Series, Exponential and Logarithmic Functions

Unit Overview
In this unit, you will study arithmetic and geometric sequences and series and their applications. You will also extend your study of exponential functions and investigate logarithmic functions and equations.

Academic Vocabulary
Add these words and others that you encounter in this unit to your vocabulary notebook.
- exponential function
- extraneous solution
- increasing/decreasing
- logarithm
- sequence
- series

Essential Questions
- How are functions that grow at a constant rate distinguished from those that do not grow at a constant rate?
- How are logarithmic and exponential equations used to model real-world problems?

Embedded Assessments
This unit has three embedded assessments, following Activities 2.2, 2.4, and 2.6. By completing these embedded assessments, you will demonstrate your understanding of arithmetic and geometric sequences and series, as well as exponential and logarithmic functions and equations.

- Embedded Assessment 1
  Sequences and Series p. 95

- Embedded Assessment 2
  Exponential Functions and Common Logarithms p. 119

- Embedded Assessment 3
  Exponential and Logarithmic Equations p. 137
Write your answers on notebook paper or grid paper. Show your work.

1. Describe the pattern displayed by 1, 2, 5, 10, 17, …

2. Give the next 3 terms of the sequence 0, −2, 1, −3, …. 

3. Draw Figure 4, using the pattern below. Then explain how you would create any figure in the pattern.

4. Simplify each expression.
   a. \( \left( \frac{6x^2}{y^3} \right)^2 \)
   b. \((2a^2b)(3b^3)\)
   c. \(\frac{10a^{12}b^6}{5a^3b^{-2}}\)

5. Evaluate the expression.
   \(\frac{3^{127}}{3^{123}}\)

6. Express the product in scientific notation.
   \((2.9 \times 10^3)(3 \times 10^2)\)

7. Solve the equation for \(x\).
   \(19 = -8x + 35\)

8. Write a function \(C(t)\) to represent the cost of a taxicab ride, where the charge includes a fee of $2.50 plus $0.50 for each tenth of a mile \(t\). Then give the slope and \(y\)-intercept of the graph of the function.
Hydrocarbons are the simplest organic compound, containing only carbon and hydrogen atoms. Hydrocarbons that contain only one pair of electrons between two atoms are called alkanes. Alkanes are valuable as clean fuels because they burn to form water and carbon dioxide. The number of carbon and hydrogen atoms in a molecule of the first six alkanes is shown in the table below.

<table>
<thead>
<tr>
<th>Alkane</th>
<th>Carbon Atoms</th>
<th>Hydrogen Atoms</th>
</tr>
</thead>
<tbody>
<tr>
<td>methane</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>ethane</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>propane</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>butane</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>pentane</td>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>hexane</td>
<td>6</td>
<td>14</td>
</tr>
</tbody>
</table>

1. Plot the data in the table on a graph that you make using the grid in the My Notes space. Write a function \( f \), where \( f(n) \) is the number of hydrogen atoms in an alkane with \( n \) carbon atoms. Describe the domain of the function.

Any function where the domain is a set of positive consecutive integers forms a sequence. The values in the range of the function are the terms of the sequence. When naming a term in a sequence, subscripts are used rather than traditional function notation. For example, the first term in a sequence would be called \( a_1 \) rather than \( f(1) \).

Consider the sequence \( \{4, 6, 8, 10, 12, 14\} \) formed by the number of hydrogen atoms in the first six alkanes.

2. What is \( a_1 \)? What is \( a_3 \)?

3. Find the differences \( a_2 - a_1, a_3 - a_2, a_4 - a_3, a_5 - a_4, \) and \( a_6 - a_5 \). Sequences like the one above are called arithmetic sequences. An arithmetic sequence is a sequence in which the difference of consecutive terms is a constant. The constant difference is called the common difference and is usually represented by \( d \).
ACTIVITY 2.1

Arithmetic Sequences and Series

Arithmetic Alkanes

My Notes

Math Tip
In a sequence $a_{n+1}$ is the term that follows $a_n$.

CONNECT TO HISTORY
In Try These A is a famous sequence known as the Fibonacci sequence. Find out more about this interesting sequence. You can find its pattern in beehives, pine cones, and flowers.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern

4. Use $a_n$ and $a_{n+1}$ to write a general expression for the common difference $d$.

5. Determine whether the number of carbon atoms in the first six alkanes {1, 2, 3, 4, 5, 6} forms an arithmetic sequence. Explain why or why not.

TRY THESE A
Determine whether each sequence is arithmetic. If the sequence is arithmetic, state the common difference. Write your answers in the My Notes space.

<table>
<thead>
<tr>
<th>a. 3, 8, 13, 18, 23, …</th>
<th>b. 1, 2, 4, 8, 16, …</th>
</tr>
</thead>
<tbody>
<tr>
<td>c. 1, 1, 3, 5, 8, …</td>
<td>d. 20, 17, 14, 11, 8, …</td>
</tr>
</tbody>
</table>

The terms in an arithmetic sequence can be written as the sum of the first term and a multiple of the common difference.

6. Complete the blanks for the sequence {4, 6, 8, 10, 12, 14, …} formed by the number of hydrogen atoms.

\[
\begin{align*}
a_1 &= \_\_\_\_ \quad & d &= \_\_\_\_ \\
a_2 &= 4 + \_\_\_\_ \cdot 2 = 6 \\
a_3 &= 4 + \_\_\_\_ \cdot 2 = 8 \\
a_4 &= 4 + \_\_\_\_ \cdot 2 = \_\_\_\_ \\
a_5 &= 4 + \_\_\_\_ \cdot 2 = \_\_\_\_ \\
a_6 &= 4 + \_\_\_\_ \cdot 2 = \_\_\_\_ \\
a_{10} &= 4 + \_\_\_\_ \cdot 2 = \_\_\_\_ \\
\end{align*}
\]
7. Write a general expression in terms of \( n \) for finding the number of hydrogen atoms in an alkane molecule with \( n \) carbon atoms.

8. Show how to use the expression you wrote in Item 7 to find the number of hydrogen atoms in decane, the alkane with 10 carbon atoms.

9. Find the number of carbon atoms in a molecule of an alkane with 38 hydrogen atoms.

10. Use \( a_1, d, \) and \( n \) to write a formula for \( a_n \), the \( n \)th term of any arithmetic sequence.

11. Use the formula from Item 10 to find the specified term in each arithmetic sequence.
   
   a. Find the 40th term when \( a_1 = 6 \) and \( d = 3 \).
   
   b. Find the 30th term of the arithmetic sequence 37, 33, 29, 25, …
TRY THESE B

Use the formula for the $n$th term of an arithmetic sequence to solve $a–d$.
Write your answers in the My Notes space. Show your work.

a. Find the 50th term when $a_1 = 7$ and $d = -2$.

b. Find the 28th term of the arithmetic sequence 3, 7, 11, 15, 19,…

c. Which term in the arithmetic sequence 15, 18, 21, 24, … is equal to 72?

d. Find $a_1$ and $d$ when $a_8 = 9$ and $a_{13} = 24$.

12. Show that the expressions for $a_n$ in Item 7 and $f(n)$ in Item 1 are equivalent.

A **series** is the sum of the terms in a sequence. The sum of the first $n$ terms of a series is the $n$th **partial sum** of the series and is denoted by $S_n$.

13. Consider the arithmetic sequence \{4, 6, 8, 10, 12, 14, 16, 18\}.

a. Find $S_4$.

b. Find $S_5$.

c. Find $S_8$.

d. How does $a_1 + a_8$ compare to $a_2 + a_7$, $a_3 + a_6$, and $a_4 + a_5$?

e. Explain how to find $S_8$ using the value of $a_1 + a_8$. 

ACADEMIC VOCABULARY

series
14. Consider the arithmetic series $1 + 2 + 3 + \ldots + 98 + 99 + 100$.

   a. How many terms are in this series?

   b. If all the terms in this series are paired as shown below, how many pairs will there be?

      $1 + 2 + 3 + 4 + \ldots + 97 + 98 + 99 + 100$

   c. What is the sum of each pair of numbers?

   d. Find the sum of the series. Explain how you arrived at the sum.

15. Consider the arithmetic series $a_1 + a_2 + a_3 + \ldots + a_{n-2} + a_{n-1} + a_n$.

      $a_1 + a_2 + a_3 + \ldots + a_{n-2} + a_{n-1} + a_n$

   a. Write an expression for the number of pairs of terms in this series.

   b. Write a formula for $S_n$, the partial sum of the arithmetic series.
16. Show how to use the formula from Item 15b to find each partial sum of the arithmetic sequence {4, 6, 8, 10, 12, 14, 16, 18}. Compare your results to your answers in Item 13.

a. \( S_4 \)

b. \( S_5 \)

c. \( S_8 \)

17. A second form of the formula for finding the partial sum of an arithmetic series is \( S_n = \frac{n}{2} [2a_1 + (n - 1) d] \). Show how to derive this, using the formula from Item 15b and the \( n^{th} \) term formula, \( a_n = a_1 + (n - 1) d \), from Item 10.

18. Show how to use the formula \( S_n = \frac{n}{2} [2a_1 + (n - 1) d] \) to find the indicated partial sum of each arithmetic series.

a. \( 3 + 8 + 13 + 18 + \ldots; S_{20} \)

b. \( -2 - 4 - 6 - 8 - \ldots; S_{18} \)
TRY THESE C

Find the indicated sum of each arithmetic series. Write your answers in the My Notes space. Show your work.

a. Find $S_8$ for the arithmetic series with $a_1 = 5$ and $a_8 = 40$.

b. $12 + 18 + 24 + 30 + \ldots; S_{10}$

c. $30 + 20 + 10 + 0 + \ldots; S_{25}$

**Summation**, or **sigma notation** ($\sum$), is a shorthand notation used to express the sum of a series. The expression $\sum_{n=1}^{4} (2n + 5)$ is read “the sum from $n = 1$ to $n = 4$ of $2n + 5$.”

To expand the series to show the terms of the series, substitute 1, 2, 3 and 4 into the expression for the general term. To find the sum of the series, add the terms.

$$\sum_{n=1}^{4} (2n + 5) = (2 \cdot 1 + 5) + (2 \cdot 2 + 5) + (2 \cdot 3 + 5) + (2 \cdot 4 + 5)$$

$$= 7 + 9 + 11 + 13 = 40$$

19. Write the first four terms and the last term in the series $\sum_{n=1}^{100} (2n - 3)$. Then find the indicated sum.

**Math Tip**

To find the first term in a series written in sigma notation, substitute the value of the lower limit into the expression for the general term.

To find subsequent terms, substitute consecutive integers that follow the lower limit, stopping at the upper limit.
ACTIVITY 2.1  Arithmetic Sequences and Series

Arithmetic Alkanes

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Think/Pair/Share

My Notes

20. Write the sum of the first 10 terms of $80 + 75 + 70 + 65 + \ldots$ using sigma notation.

21. Summarize the following formulas for an arithmetic series.

- Common difference: $d = \ldots$
- $n^{th}$ term: $a_n = \ldots$
- Sum of first $n$ terms: $S_n = \ldots$

or

$S_n = \ldots$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Determine whether each sequence is arithmetic. If the sequence is arithmetic, state the common difference.
   a. $4, 5, 7, 10, \ldots$
   b. $5, 7, 9, 11, \ldots$
   c. $12, 9, 6, 3, \ldots$
   d. $7, 7.5, 8, 8.5, \ldots$

2. Find the indicated term of each arithmetic sequence.
   a. $a_1 = 4, d = 5; a_{15}$
   b. $14 + 18 + 22 + 26 + \ldots; S_{12}$
   c. $\sum_{n=1}^{15} (3n - 1)$

3. Find the indicated partial sum of each arithmetic series.
   a. $a_1 = 4, d = 5; S_{10}$
   b. $14 + 18 + 22 + 26 + \ldots; S_{12}$
   c. $\sum_{n=1}^{15} (3n - 1)$

4. A store puts boxes of canned goods into a stacked display. There are 20 boxes in the bottom layer. Each layer has two fewer boxes than the layer below it. There are 5 layers of boxes. How many boxes are in the display? Use a formula to answer the question.

5. MATHEMATICAL REFLECTION  How does the common difference of an arithmetic sequence relate to finding the partial sum of an arithmetic sequence?
Meredith is designing a mural for an outside wall of a warehouse that is being converted into the Taylor Modern Art Museum. The mural is 32 feet wide by 31 feet high. The design consists of squares in 5 different sizes that are painted black or white as shown in the figure below.

1. Let Square 1 be the largest size and Square 5 be the smallest size. For each size, record the length of the side, the number of squares of that size in the design, and the area of the square.

<table>
<thead>
<tr>
<th>Square #</th>
<th>Side of Square (ft)</th>
<th>Number of Squares</th>
<th>Area of Square (ft²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. Refer to the table in Item 1.
   a. Describe any patterns that you notice in the table.

b. Each column of numbers forms a sequence of numbers. List the four sequences that you see in the columns of the table.

c. Are any of those sequences arithmetic? Why or why not?

A geometric sequence is a sequence in which the ratio of consecutive terms is a constant. The constant is called the common ratio and is denoted by $r$.

3. Consider the sequences in Item 2b.
   a. List those sequences that are geometric.

   b. State the common ratio for each geometric sequence.
4. Use $a_n$ and $a_{n+1}$ to write a general expression for the common ratio $r$. 

5. Consider the sequences in the columns of the table in Item 1 that are labeled Square # and Side of Square. Use the grid in the My Notes section to draw a graph.

   a. Plot the Square # sequence by plotting the ordered pairs (term number, square number).

   b. Using another color or symbol, plot the Side of Square sequence by plotting the ordered pairs (term number, side of square).

   c. Is either sequence a linear function? Explain why or why not.

TRY THESE A

Determine whether each sequence is arithmetic, geometric, or neither. If the sequence is arithmetic, state the common difference. If it is geometric, state the common ratio. Write your answers in the My Notes space.

a. 3, 9, 27, 81, 243, …

b. 1, −2, 4, −8, 16, …

c. 4, 9, 16, 25, 36, …

d. 25, 20, 15, 10, 5, …
The terms in a geometric sequence can be written as the product of the first term and a power of the common ratio.

6. For the geometric sequence \{4, 8, 16, 32, 64, \ldots\}, identify \(a_1\) and \(r\). Then fill in the missing exponents and blanks.

\[ a_1 = \ldots \quad r = \ldots \]

\[ a_2 = 4 \cdot 2 = 8 \]
\[ a_3 = 4 \cdot 2 = 16 \]
\[ a_4 = 4 \cdot 2 = \ldots \]
\[ a_5 = 4 \cdot 2 = \ldots \]
\[ a_6 = 4 \cdot 2 = \ldots \]
\[ a_{10} = 4 \cdot 2 = \ldots \]

7. Use \(a_1\), \(r\), and \(n\) to write a formula for the \(n^{\text{th}}\) term of any geometric sequence.

8. Use the formula from Item 7 to find the indicated term in each geometric sequence.

a. \(1, 2, 4, 8, 16, \ldots; a_{16}\)

b. \(4096, 1024, 256, 64, \ldots; a_9\)
The sum of the terms of a geometric sequence is a geometric series. The sum of a finite geometric series where \( r \neq 1 \) is given by these formulas:

\[
S_n = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \ldots + a_1 r^{n-1}
\]

\[
S_n = a_1 \left( \frac{1 - r^n}{1 - r} \right)
\]

9. To derive the formula, Step 1 requires multiplying the equation of the sum by \(-r\). Follow the remaining steps on the left to complete the derivation of the sum formula.

\[
S_n = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \ldots + a_1 r^{n-1}
\]

\[
-rS_n = -a_1 r - a_1 r^2 - a_1 r^3 - \ldots - a_1 r^{n-1} - a_1 r^n
\]

**Step 2**
Combine terms on each side the equation.

**Step 3**
Factor out \( S_n \) on the left side of the equation and factor out \( a_1 \) on the right.

**Step 4**
Solve for \( S_n \).

10. Find the total of the Area of Square column in the table in Item 1. Then use the formula developed in Item 9 to find the total area and show that the result is the same.

**TRY THESE B**
Find the indicated sum of each geometric series. Write your answers on notebook paper. Show your work.

**a.** Find \( S_5 \) for the geometric series with \( a_1 = 5 \) and \( r = 2 \).

**b.** \( 256 - 64 + 16 - 4 + \ldots; S_6 \)

**c.** \( \sum_{n=1}^{10} 2 \cdot 3^{n-1} \)

**Math Tip**
Recall that sigma notation is a shorthand notation for a series. For example:

\[
\sum_{n=1}^{3} 8 \cdot 2^{n-1}
\]

\[
= 8(2)^{1-1} + 8(2)^2 - 1 + 8(2)^3 - 1
\]

\[
= 8 \cdot 1 + 8 \cdot 2 + 8 \cdot 4
\]

\[
= 8 + 16 + 32
\]

\[
= 56
\]
Recall from Activity 2.1 that the sum of the first \( n \) terms of a series is a partial sum. For some geometric series, the partial sums, \( S_1, S_2, S_3, S_4, \ldots \) form a sequence with terms that approach a limiting value. The limiting value is called the sum of the infinite geometric series.

To illustrate the concept of an infinite sum of the geometric series that sums to a specific number, follow these steps.

- Start with a square piece of paper, and let it represent one whole.
- Cut the paper in half, place one piece of the paper on your desk, and keep the other piece of paper in your hand. The paper on your desk represents the first partial sum of the series, \( S_1 = \frac{1}{2} \).
- Cut the paper in your hand in half again, adding one of the pieces to the paper on your desk and keeping the other piece in your hand. The paper on your desk now represents the second partial sum.
- Repeat this process as many times as you are able.

11. Each time you add a piece of paper to your desk, the paper represents the next term in the geometric series.

- As you continue the process of placing half of the remaining paper on your desk, what happens to the amount of paper on your desktop?
11. (continued)

b. Fill in the blanks to complete the partial sums for the infinite geometric series represented by the pieces of paper on your desk.

\[ S_1 = \frac{1}{2} \]

\[ S_2 = \frac{1}{2} + \_\_ = \_\_ \]

\[ S_3 = \frac{1}{2} + \_\_ + \_\_ = \_\_ \]

\[ S_4 = \frac{1}{2} + \_\_ + \_\_ + \_\_ = \_\_ \]

\[ S_5 = \frac{1}{2} + \_\_ + \_\_ + \_\_ + \_\_ = \_\_ \]

\[ S_6 = \frac{1}{2} + \_\_ + \_\_ + \_\_ + \_\_ + \_\_ = \_\_ \]

c. On the grid in the My Notes space, draw a graph and plot the first six partial sums.

d. Do the partial sums appear to be approaching a limiting value? If so, what does the value appear to be?

**CONNECT TO AP**

An infinite series whose partial sums continually get closer to a specific number is said to converge, and that number is called the sum of the infinite series.
12. Consider the geometric series $2 + 4 + 8 + 16 + 32 + \ldots$.
   
   a. List the first five partial sums for this series.
   
   b. Do these partial sums appear to have a limiting value?
   
   c. Does the series appear to have an infinite sum? If so, what does the sum appear to be? If not, why not?

13. Consider the geometric series $3 - 1 + \frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81} + \frac{1}{243} - \ldots$
   
   a. List the first seven partial sums for this series.
   
   b. Do these partial sums appear to have a limiting value?
   
   c. Does the series have an infinite sum? If so, what does the sum appear to be? If not, why not?

**Writing Math**

You can write an infinite sum by using summation, or sigma notation, and using an infinity symbol for the upper limit. For example,

$$\sum_{n=1}^{\infty} 3\left(-\frac{1}{3}\right)^{n-1} = 3 - 1 + \frac{1}{3} - \ldots$$
An infinite geometric series \( \sum_{n=0}^{\infty} a_n r^n \) converges to the sum \( S = \frac{a_1}{1-r} \) if and only if \( |r| < 1 \), or \(-1 < r < 1\). If \( |r| \geq 1 \), the infinite sum does not exist.

14. Consider the three series from Items 11–13. Decide whether the formula for the sum of an infinite geometric series applies. If so, use it find the sum. Compare the results to your previous answers.

a. \( \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots \)

b. \( 2 + 4 + 6 + 8 + 10 + \ldots \)

c. \( 3 - 1 + \frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81} + \frac{1}{243} - \ldots \)

15. Consider the arithmetic series \( 2 + 5 + 8 + 11 + \ldots \).

a. Find the first four partial sums of the series.

b. Do these partial sums appear to have a limiting value?
15. (continued)
   c. Does the arithmetic series appear to have an infinite sum? Explain.

16. Summarize the following formulas for a geometric series.

   - common ratio \( r = \) _____________
   - \( n^{th} \) term \( a_n = \) _____________
   - Sum of first \( n \) terms \( S_n = \) _____________
   - Infinite sum \( S = \) _____________

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Determine whether each sequence is arithmetic, geometric or neither. If the sequence is arithmetic, state the common difference and if it is geometric state the common ratio.
   a. 3, 5, 7, 9, 11, …
   b. 5, 15, 45, 135, …
   c. 6, –4, \( \frac{8}{3} \), \( \frac{16}{9} \), …
   d. 1, 2, 4, 7, 11, …

2. Find the indicated term of each geometric series.
   a. \( a_1 = -2, r = 3; a_9 \)
   b. \( a_1 = 1024, r = \frac{1}{2}; a_{12} \)

3. Find the indicated partial sum of each geometric series.
   a. 1, –3, 9, –27, …; \( S_7 \)
   a. \( \frac{1}{625}, \frac{1}{125}, \frac{1}{25}, \frac{1}{5}, \ldots; S_9 \)

4. During a 10-week summer promotion, a baseball team is letting all spectators enter their names in a weekly drawing each time they purchase a game ticket. Once a name is in the drawing, it remains in the drawing unless it is chosen as a winner. Since the number of names in the drawing increases each week, so does the prize money. The first week of the contest the prize amount is $10, and it doubles each week.
   a. What is the prize amount in the 4th week of the contest? In the 10th week?
   b. What is the total amount of money given away during the entire promotion?

5. Find the infinite sum if it exists, or tell why it does not exist.
   a. \( 18 - 9 + \frac{9}{2} - \frac{9}{4} + \ldots \)
   b. \( 729 + 486 + 324 + 216 + \ldots \)
   c. \( 81 + 108 + 144 + 192 + \ldots \)

6. True or false? No arithmetic series except \( 0 + 0 + 0 + 0 \ldots \) has an infinite sum. Explain your thinking.
Sequences and Series

THE CHESSBOARD PROBLEM

In a classic math problem a king wants to reward a knight who has rescued him from an attack. The king gives the knight a chessboard and plans to place money on each square. He gives the knight two options. Option 1 is to place a thousand dollars on the first square, two thousand on the second square, three thousand on the third square, and so on. Option 2 is to place one penny on the first square, two pennies on the second, four on the third, and so on.

Think about which offer sounds better and then answer these questions. Write your answers on notebook paper. Show your work.

1. List the first five terms in the sequences formed by the given options. Identify each sequence as arithmetic, geometric, or neither.
   a. Option 1
   b. Option 2

2. For each option, write a rule that tells how much money is placed on the $n$th square of the chessboard and a rule that tells the total amount of money placed on squares one through $n$.
   a. Option 1
   b. Option 2

3. Find the amount of money placed on the 20th square of the chessboard and the total amount placed on squares 1 through 20 for each option.
   a. Option 1
   b. Option 2

4. There are 64 squares on a chessboard. Find the total amount of money placed on the chessboard for each option.
   a. Option 1
   b. Option 2

5. Which gives the better reward, Option 1 or Option 2? Explain why.
# Sequences and Series

**THE CHESSBOARD PROBLEM**

Use after Activity 2.2.

<table>
<thead>
<tr>
<th>Math Knowledge #3a, b; 4a, b</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student:</td>
<td>The student:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Finds the total amount</td>
<td>• Finds the</td>
<td></td>
<td></td>
</tr>
<tr>
<td>of money placed on the 20th</td>
<td>amount of</td>
<td></td>
<td></td>
</tr>
<tr>
<td>square and the total</td>
<td>money placed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>amount placed on squares</td>
<td>on the 20th</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 through 20, based on</td>
<td>square and the</td>
<td></td>
<td></td>
</tr>
<tr>
<td>the student’s rules.</td>
<td>total amount</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3a, b)</td>
<td>placed on</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Finds the total amount</td>
<td>squares 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>placed on the chessboard</td>
<td>through 20 for</td>
<td></td>
<td></td>
</tr>
<tr>
<td>based on the student’s</td>
<td>only one of the</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rules. (4a, b)</td>
<td>options.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Finds the</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>total amount</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>placed on the</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>chessboard for</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>only one of the</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>options.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem Solving #1a, b</th>
<th>Problem Solving #1a, b</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student lists the first five correct terms in both sequences and correctly identifies both types of sequence. (1a, b)</td>
<td>The student lists the first five correct terms in both of the sequences but correctly identifies only one type of sequence.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Representations #2a, b</th>
<th>Representations #2a, b</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student writes the correct rule. (2a, b)</td>
<td>The student writes only one correct rule.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Communication #5</th>
<th>Communication #5</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student gives a complete explanation for the option that gives the better reward. (5)</td>
<td>The student gives an incomplete explanation for the option that gives the better reward.</td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Ramon Hall, a graphic artist, needs to make several different-sized draft copies of an original design. His original graphic design sketch is contained within a rectangle with a width of 4 cm and a length of 6 cm. Using the office copy machine, he magnifies the original 4 cm \( \times \) 6 cm design to 120% of the original design size, and calls this his first draft. Ramon's second draft results from magnifying the first draft to 120% of its new size. Each new draft is 120% of the previous draft.

1. Complete the table with the dimensions of Ramon's first five draft versions, showing all decimal places.

<table>
<thead>
<tr>
<th>Number of Magnifications</th>
<th>Width (cm)</th>
<th>Length (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. The resulting draft for each number of 120% magnifications has a unique width and a unique length. Thus, there is a functional relationship between the number of magnifications \( n \) and the resulting width \( W \). There is also a functional relationship between the number of magnifications \( n \) and the resulting length \( L \). Make a graph using the grid in the My Notes space and the data from the table in Item 1.

   a. Plot the ordered pairs \((n, W)\).

   b. Use a different color or symbol to plot the ordered pairs \((n, L)\).

3. Examine the data and graphs in Items 1 and 2. Do these functions appear to be linear? Explain why or why not.
4. Explain why each table contains data that can be represented by a linear function. Write an equation to show the linear relationship between $x$ and $y$.

|   | $x$  | $y$  \\
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>$-3$</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>$-1$</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>$-1$</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$-4$</td>
</tr>
</tbody>
</table>

$y = \underline{\phantom{0000000000000}}$

|   | $x$  | $y$  \\
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>b.</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>26</td>
<td>35</td>
</tr>
</tbody>
</table>

$y = \underline{\phantom{0000000000000}}$

5. Consider the data in the table below.

|   | $x$  | $y$  \\
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.5</td>
</tr>
</tbody>
</table>

a. Can the data in the table be represented by a linear function? Explain why or why not.

b. Describe any patterns that you see in the consecutive $y$-values.
6. Consider the data in the table created in Item 1.

   a. Copy the width values into the second row of the table below.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. How does the relationship of the data in this table compare to the relationship of the data in the table in Item 5?

The data in the tables in Items 5 and 6 were generated by functions that are exponential. In the special case when the change in the input variable \(x\) is constant, the output variable \(y\) of an exponential function changes by a multiplicative constant. For example, in the table in Item 5, the increase in the consecutive \(x\)-values results from repeatedly adding 1, while the decrease in \(y\)-values results from repeatedly multiplying by the constant \(\frac{1}{2}\), known as the exponential decay factor.

7. In the table in Item 6, the increase in the \(x\)-values results from repeatedly adding 1.

   a. What is the exponential growth factor?

   b. What is the growth rate and how is it related to the exponential growth factor?

ACADEMIC VOCABULARY

An exponential function is a function of the form \(f(x) = a \cdot b^x\), where \(a\) and \(b\) are constants, \(x\) is the domain, \(f(x)\) is the range, and \(a \neq 0\), \(b > 0\), \(b \neq 1\).

MATH TERMS

In an exponential function, the constant multiplier, or scale factor, is called an exponential decay factor when the constant is less than 1. When the constant is greater than 1, it is called an exponential growth factor.

Math Tip

To compare change in size, you could also use the growth rate, or percent increase. This is the percent that is equal to the ratio of the increase amount to the original amount.
8. You can write an equation for the exponential function relating \( W \) and \( n \).

a. Complete the table below to show the calculations used to find the width of each magnification.

<table>
<thead>
<tr>
<th>Number of Magnifications</th>
<th>Calculation to Find Width (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>4(1.2)</td>
</tr>
<tr>
<td>2</td>
<td>4(1.2)(1.2)</td>
</tr>
<tr>
<td>3</td>
<td>4(1.2)(1.2)(1.2)</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td></td>
</tr>
</tbody>
</table>

b. Write a function that expresses the resulting width \( W \) after \( n \) magnifications of 120%.

c. Use the function in Part b to find the width of the 11th magnification.
The general form of an exponential function is \( f(x) = a(b^x) \), where \( a \) and \( b \) are constants, and \( a \neq 0, b > 0, b \neq 1 \).

9. For the exponential function written in Item 8b, identify the value of the parameters \( a \) and \( b \). Then explain their meaning in terms of the problem situation.

10. Starting with Ramon’s original 4 cm \( \times \) 6 cm rectangle containing his graphic design, write an exponential function that expresses the resulting length \( L \) after \( n \) magnifications of 120%.

Ramon decides to print five different reduced draft copies of his original design rectangle. Each one will be reduced to 90% of the previous size.

11. Complete the table below to show the dimensions of the first five draft versions. Include all decimal places.

<table>
<thead>
<tr>
<th>Number of Reductions</th>
<th>Width (cm)</th>
<th>Length (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

12. Write the exponential decay factor and the decay rate for the data in the table in Item 11.

To compare change in size, you could also use the decay rate, or percent decrease. This is the percent that is equal to the ratio of the decrease amount to the original amount.
13. Use the data in the table in Item 11.

   a. Write an exponential function that expresses the width \( w \) of a reduction in terms of \( n \), the number of reductions performed.

   b. Write an exponential function that expresses the length \( l \) of a reduction in terms of \( n \), the number of reductions performed.

   c. Use the functions to find the dimensions of the design if the original design undergoes ten reductions.

14. Graph the functions \( y = 6(1.2)^x \) and \( y = 6(0.9)^x \) on a graphing calculator or other graphing utility. Sketch the results on the grid in the My Notes space.

15. Determine the domain and range for each function.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a. \ y = 6(1.2)^x )</td>
<td></td>
</tr>
<tr>
<td>( b. \ y = 6(0.9)^x )</td>
<td></td>
</tr>
</tbody>
</table>

A function is said to increase if the \( y \)-values increase as the \( x \)-values increase. A function is said to decrease if the \( y \)-values decrease as the \( x \)-values increase.

16. Describe each function as increasing or decreasing.

   a. \( y = 6(1.2)^x \)

   b. \( y = 6(0.9)^x \)
The end behavior of a graph describes the y-values of the function as \( x \) increases without bound and as \( x \) decreases without bound. If the end behavior approaches some constant \( a \), then the graph of the function has a horizontal asymptote at \( y = a \).

When \( x \) increases without bound, the values of \( x \) approach positive infinity, \( \infty \). When \( x \) decreases without bound, the values of \( x \) approach negative infinity, \( -\infty \).

17. Describe the end behavior of each function as \( x \) approaches \( \infty \). Write the equation for any horizontal asymptotes.

   \( a. \quad y = 6(1.2)^x \)
   
   \( b. \quad y = 6(0.9)^x \)

18. Describe the end behavior of each function as \( x \) approaches \( -\infty \). Write the equation for any horizontal asymptotes.

   \( a. \quad y = 6(1.2)^x \)
   
   \( b. \quad y = 6(0.9)^x \)

19. Identify any \( x \)- or \( y \)-intercepts of each function.

   \( a. \quad y = 6(1.2)^x \)
   
   \( b. \quad y = 6(0.9)^x \)

20. Consider how the parameters \( a \) and \( b \) affect the graph of the general exponential function \( f(x) = a(b)^x \). Use a graphing calculator to graph \( f \) for various values of \( a \) and \( b \).

   \( a. \quad \text{When does the function increase?} \)
   
   \( b. \quad \text{When does the function decrease?} \)
   
   \( c. \quad \text{What determines the} \ y \text{-intercept of the function?} \)
   
   \( d. \quad \text{State any horizontal asymptotes of the function.} \)
You can use transformations of the graph of the function \( f(x) = b^x \) to graph functions of the form \( g(x) = a(b)^{x-c} + d \), where \( a \) and \( b \) are constants, and \( a \neq 0, b > 0, b \neq 1 \). Rather than having a single parent graph for all exponential functions, there is a different parent graph for each base \( b \).

21. Graph the parent graph \( f \) and the function \( g \) by applying the correct vertical stretch, shrink, and/or reflection over the \( x \)-axis. Write a description of each transformation.

a. \( f(x) = \left( \frac{1}{2} \right)^x \)
   \[ g(x) = 4 \left( \frac{1}{2} \right)^x \]

b. \( f(x) = 3^x \)
   \[ g(x) = -\frac{1}{2}(3)^x \]
ACTIVITY 2.3 continued

Exponential Functions and Graphs

Sizing Up the Situation

SUGGESTED LEARNING STRATEGIES: Create Representations, Quickwrite

22. Sketch the parent graph \( f \) and the graph of \( g \) by applying the correct horizontal or vertical translation. Write a description of each transformation and give the equations of any asymptotes.

a. \( f(x) = 2^x \)

\[ g(x) = 2^{(x - 3)} \]

You can use a graphing calculator to approximate the range values when the \( x \)-coordinates are not integers. For \( f(x) = 2^x \), use a calculator to find:

\[ f\left(\frac{1}{2}\right) \text{ and } f(\sqrt{3}) \]

\[ 2^{\frac{1}{2}} \approx 1.414 \]

\[ 2^{\sqrt{3}} \approx 3.322 \]

Then use a graphing calculator to verify that the points \( \left(\frac{1}{2}, 2^{\frac{1}{2}}\right) \) and \( \left(\sqrt{3}, 2^{\sqrt{3}}\right) \) lie on the graph of \( f(x) = 2^x \).

b. \( f(x) = \left(\frac{1}{3}\right)^x \)

\[ g(x) = \left(\frac{1}{3}\right)^x - 2 \]
23. Describe how each function results from transforming a parent graph of the form \( f(x) = b^x \). Then sketch the parent graph and the given function on the same axes. State the domain and range of each function. Give the equations of any asymptotes.

a. \( g(x) = 3^{x + 4} + 1 \)

\[
\begin{array}{c|c|c|c|c|c|c}
\hline
x & -7 & -6 & -5 & -4 & -3 & -2 \\
\hline
y & -1 & 0 & 1 & 2 & 3 & 4 \\
\hline
\end{array}
\]

b. \( g(x) = 2\left(\frac{1}{3}\right)^x - 4 \)

\[
\begin{array}{c|c|c|c|c|c|c}
\hline
x & -7 & -6 & -5 & -4 & -3 & -2 \\
\hline
y & -1 & 0 & 1 & 2 & 3 & 4 \\
\hline
\end{array}
\]
23. (continued)

- **c.** \( g(x) = \frac{1}{2}(4)^{-x} - 2 \)

- **d.** \( g(x) = -3(2)^{x-6} + 5 \)
ACTIVITY 2.3 continued

Exponential Functions and Graphs
Sizing Up the Situation

SUGGESTED LEARNING STRATEGIES: Quickwrite, Group Presentation

24. Explain how the parameters $a$, $c$, and $d$ transform the parent graph $f(x) = b^x$ to produce a graph of the function $g(x) = a(b)^{x-c} + d$.

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper or grid paper. Show your work.

1. Ida paints violets onto porcelain plates. She paints a spiral that is a sequence of violets, the size of each consecutive violet being a fraction of the size of the preceding violet. The table below shows the width of the first three violets in the continuing pattern.

<table>
<thead>
<tr>
<th>Violet Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width (in cm)</td>
<td>4</td>
<td>3.2</td>
<td>2.56</td>
</tr>
</tbody>
</table>

a. Is Ida’s shrinking violet pattern an example of an exponential function? Explain.
b. Find the width of the fourth and fifth violets in the sequence.
c. Write an equation to express the size of the smallest violet in terms of the number of violets on the plate.
d. If a plate has a total of 10 violets, explain two different ways to determine the size of the smallest violet.

2. For each exponential function, state the domain and range, whether the function increases or decreases, and the $y$-intercept.
   a. $y = 2(4)^x$
   b. $y = 3\left(\frac{1}{2}\right)^x$
   c. $y = -(0.3)^x$
   d. $y = -3(5.2)^x$

3. Describe how each function results from transforming a parent graph of the form $f(x) = b^x$. Then sketch the parent graph and the given function on the same axes. State the domain and range of each function and give the equations of any asymptotes.
   a. $g(x) = 2^{x+3} - 4$
   b. $g(x) = -3\left(\frac{1}{2}\right)^x + 2$
   c. $g(x) = \frac{1}{2}(3)^{x+3} - 4$

4. **Mathematical Reflection** Which function “grows” faster, a linear function or an exponential function? Explain your reasoning.
Common Logarithms and Their Properties

Earthquakes and Richter Scale

SUGGESTED LEARNING STRATEGIES: Summarize/Paraphrase/Retell, Create Representations

In 1935, Charles F. Richter developed the Richter magnitude test scale to compare the size of earthquakes. The Richter scale is based on the amplitude of the seismic waves recorded on seismographs at various locations after being adjusted for distance from the epicenter of the earthquake.

Richter assigned a magnitude of 0 to a standard earthquake, whose amplitude on a seismograph is 1 micron, or $10^{-4}$ cm. According to the Richter scale, a magnitude-1.0 earthquake causes 10 times the ground motion of a standard earthquake. A magnitude-2.0 earthquake causes 10 times the ground motion of a magnitude-1.0 earthquake. This pattern continues as the magnitude of the earthquake increases.

1. How does the ground motion caused by earthquakes of these magnitudes compare?
   a. magnitude-5.0 earthquake compared to magnitude 4.0
   b. magnitude-4.0 earthquake compared to magnitude 1.0
   c. magnitude-4.0 earthquake compared to a standard earthquake whose magnitude is 0

2. The table below describes the effects of different magnitude earthquakes. Complete the table to show how many times as great the ground motion is when caused by each earthquake as compared to the standard earthquake.

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>Ground Motion Compared to Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>10</td>
</tr>
<tr>
<td>2.0</td>
<td>100</td>
</tr>
<tr>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>5.0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>Ground Motion Compared to Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.0</td>
<td></td>
</tr>
<tr>
<td>7.0</td>
<td></td>
</tr>
<tr>
<td>8.0</td>
<td></td>
</tr>
<tr>
<td>9.0</td>
<td></td>
</tr>
<tr>
<td>10.0</td>
<td></td>
</tr>
</tbody>
</table>

Typical Effects of Earthquakes of Various Magnitudes

1.0 Very weak, no visible damage
2.0 Not felt by humans
3.0 Often felt, usually no damage
4.0 Windows rattle, indoor items shake
5.0 Damage to poorly constructed structures, slight damage to well-designed buildings
6.0 Destructive in populated areas
7.0 Serious damage over large geographic areas
8.0 Serious damage across areas of hundreds of miles
9.0 Serious damage across areas of hundreds of miles
10.0 Extremely rare, never recorded
3. In Parts a–c, you will graph the data from Item 2. Let the horizontal axis represent the magnitude of the earthquake and the vertical axis represent the amount of ground motion caused by the earthquake as compared to a standard earthquake. Use the grid in the My Notes space or a separate sheet of grid paper.

a. Plot the data, using a grid that displays \(-10 \leq x \leq 10\) and \(-10 \leq y \leq 10\). Explain why this grid is or is not a good choice.

b. Plot the data, using a grid that displays \(-10 \leq x \leq 100\) and \(-10 \leq y \leq 100\). Explain why this grid is or is not a good choice.

c. A scale may be easier to choose if only a subset of the data is graphed. Determine an appropriate subset of the data and a scale for the graph. Draw the graph and label and scale the axes.

d. Draw a function that fits the data plotted on the graph in Part c. Write a function \(G\) for the ground motion caused compared to a standard earthquake by a magnitude-\(x\) earthquake.
Charles Richter needed a way to convert these values into more accessible numbers. He also realized that the magnitude of an earthquake is a function of ground motion caused by the earthquake, because only after the ground motion is measured by a seismograph can a magnitude be assigned to the earthquake.

4. In Item 3, the data was plotted so that the ground motion caused by the earthquake was a function of the magnitude of the earthquake.
   a. Is the ground motion a result of the magnitude of an earthquake or is the magnitude of an earthquake the result of ground motion?

   b. Based your answer to Part a, would you choose ground motion or magnitude as the independent variable of a function relating the two quantities? What would you choose as the dependent variable?

   c. Make a new graph of the data plotted Item 3c so that the magnitude of the earthquake is a function of the ground motion caused by the earthquake. Scale the axes and draw a function that fits the plotted data. Use the grid in the My Notes space or use a separate sheet of grid paper.

5. Let the function you graphed in Item 4c be \( y = M(x) \), where \( M \) is the magnitude of an earthquake that causes \( x \) times as much ground motion as a standard earthquake.

   a. State the domain and the range of the function \( y = G(x) \) from Item 3d and the function \( y = M(x) \).

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = G(x) )</td>
<td></td>
</tr>
<tr>
<td>( y = M(x) )</td>
<td></td>
</tr>
</tbody>
</table>
5. (continued)

b. In terms of the problem situation, describe the meaning of an ordered pair on the graphs of \( y = G(x) \) and \( y = M(x) \).

\[
y = G(x) \quad (__, __)
\]

\[
y = M(x) \quad (__, __)
\]

c. A portion of the graphs of \( y = G(x) \) and \( y = M(x) \) is shown on the same set of axes. Describe any patterns you observe.

\[
\begin{array}{c}
\text{G(x)} \\
\text{M(x)}
\end{array}
\]

d. What is the relationship between the functions \( G \) and \( M \)?
Common Logarithms and Their Properties
Earthquakes and Richter Scale

SUGGESTED LEARNING STRATEGIES: Close Reading, Vocabulary Organizer, Create Representations, Quickwrite

The Richter scale uses a base 10 logarithmic scale. A base 10 logarithmic scale means that when the ground motion is expressed as a power of 10, the magnitude of the earthquake is the exponent. You have seen this function \( G(x) = 10^x \), where \( x \) is the magnitude, in Item 3d.

The function \( M \) is the inverse of an exponential function \( G \) whose base is 10. The algebraic rule for \( M \) is a common logarithmic function. Write this function as \( M(x) = \log x \), where \( x \) is the ground motion compared to a standard earthquake.

6. Graph \( M(x) = \log x \) on a graphing calculator.

   a. Sketch the calculator graph on the grid in the My Notes space. Be certain to label and scale each axis.

   b. Use \( M \) to estimate the magnitude of an earthquake that causes 120,000 times the ground motion of a standard earthquake. Describe what would happen if this earthquake was centered underneath a large metropolitan city.

   c. Use \( M \) to determine the amount of ground motion caused by the 2002 magnitude-7.9 Denali earthquake.

ACADEMIC VOCABULARY

A logarithm is an exponent to which a base is raised that results in a specified value.

Math Tip

You can also write the equation \( y = \log x \) as \( y = \log_{10} x \). In the equation \( y = \log x \), 10 is understood to be the base. Just as exponential functions can have bases other than 10, logarithmic functions can also be expressed with bases other than 10.
ACTIVITY 2.4
Common Logarithms and Their Properties
Earthquakes and Richter Scale

SUGGESTED LEARNING STRATEGIES: Create Representations, Think/Pair/Share

My Notes

7. Complete the tables below to show the relationship between the exponential function base 10 and its inverse, the common logarithmic function.

<table>
<thead>
<tr>
<th>x</th>
<th>y = 10^x</th>
<th>x</th>
<th>y = log x</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10^0 = 1</td>
<td>1</td>
<td>log 1 = 0</td>
</tr>
<tr>
<td>1</td>
<td>10^1</td>
<td>10</td>
<td>10^1</td>
</tr>
<tr>
<td>2</td>
<td>10^2</td>
<td>100</td>
<td>10^2</td>
</tr>
<tr>
<td>3</td>
<td>10^3</td>
<td>1000</td>
<td>10^3</td>
</tr>
<tr>
<td>log x</td>
<td></td>
<td></td>
<td>10^x</td>
</tr>
</tbody>
</table>

Math Tip
Recall that two functions are inverses when
f(f⁻¹(x)) = f⁻¹(f(x)) = x

Math Tip
The exponent x in the equation y = 10^x is the common logarithm of y. This equation can be rewritten as log y = x.

8. Use the information in Item 7 to write a logarithmic statement for each exponential statement.

a. 10^4 = 10,000
b. 10^{-1} = \frac{1}{10}

9. Use the information in Item 7 to write each logarithmic statement as an exponential statement.

a. log 100,000 = 5
b. log \left( \frac{1}{100} \right) = -2

10. Evaluate each logarithmic expression without using a calculator.

a. log 1000
b. log \left( \frac{1}{10,000} \right)
You have already learned the properties of exponents. Logarithms also have properties.

11. Complete these three properties of exponents.

\[ a^m \cdot a^n = \] ________________

\[ \frac{a^m}{a^n} = \] ________________

\[ (a^n)^a = \] ________________

12. Use a calculator to complete the tables below. Round each answer to the nearest thousandth.

<table>
<thead>
<tr>
<th>x</th>
<th>y = \log x</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>y = \log x</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

13. Add the logarithms from the tables in Item 12 to see if you can develop a property. Find each sum and round each answer to the nearest hundredth.

\[ \log 2 + \log 3 = \] ________________

\[ \log 2 + \log 4 = \] ________________

\[ \log 2 + \log 5 = \] ________________

\[ \log 3 + \log 3 = \] ________________
14. Compare the answers in Item 13 to the tables of data in Item 12.

   a. Is there a pattern or property when these logarithms are added? If yes, explain the pattern that you have found.

   b. State the property of logarithms that you found by completing the following statement.

      \[ \log m + \log n = \] _________________

15. Explain the connection between the property of logarithms stated in Item 14 and the corresponding property of exponents in Item 11.

16. Graph \( y_1 = \log 2 + \log x \) and \( y_2 = \log 2x \) on a graphing calculator. What do you observe? Explain.

17. Make a conjecture about the property of logarithms that relates to the property of exponential equations that states the following:

\[ \frac{a^m}{a^n} = a^{m-n}. \]
18. Use the information from the tables in Item 12 to provide examples that support your conjecture in Item 17.

19. Graph \( y_1 = \log x - \log 2 \) and \( y_2 = \log \left( \frac{x}{2} \right) \) on a graphing calculator. What do you observe?

20. Make a conjecture about the property of logarithms that relates to the property of exponents that states the following: \((a^m)^n = a^{mn}\).

21. Use the information from the tables in Item 12 and the properties developed in Items 14 and 17 to support your conjecture in Item 20.

22. Graph \( y_1 = 2 \log x \) and \( y_2 = \log x^2 \) on a graphing calculator. What do you observe?

23. The logarithmic properties that you conjectured and then verified in Items 14, 17, and 20 are listed below. State each property.

   - **Product Property:** ____________________________
   - **Quotient Property:** ____________________________
   - **Power Property:** ____________________________
ACTIVITY 2.4
Common Logarithms and Their Properties
Earthquakes and Richter Scale

My Notes

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Create Representations

24. Use the properties from Item 23 to condense each expression to a single logarithm. Assume all variables are positive.
   a. \( \log x - \log 7 \)
   b. \( 2 \log x + \log y \)

25. Use the properties from Item 23 to expand each expression. Assume all variables are positive.
   a. \( \log 5xy^4 \)
   b. \( \log \frac{x}{y^3} \)

26. Condense each expression to a single logarithm. Then evaluate.
   a. \( \log 2 + \log 5 \)
   b. \( \log 5000 - \log 5 \)
   c. \( 2 \log 5 + \log 4 \)

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Let \( f(x) = 10^x \) and let \( g(x) = f^{-1}(x) \). What is the algebraic rule for \( g(x) \)?
2. Evaluate without using a calculator.
   a. \( \log 10^6 \)
   b. \( 10^{\log 4} \)
   c. \( \log 1,000,000 \)
   d. \( \log \frac{1}{100} \)
3. Write an exponential statement for each.
   a. \( \log 10 = 1 \)
   b. \( \log \frac{1}{1,000,000} = -6 \)
   c. \( \log a = b \)

4. Write a logarithmic statement for each.
   a. \( 10^7 = 10,000,000 \)
   b. \( 10^0 = 1 \)
   c. \( 10^m = n \)

5. Condense each expression to a single logarithm. Then evaluate the expression without using a calculator.
   a. \( \log 5 + \log 20 \)
   b. \( \log 3 - \log 30 \)
   c. \( 2 \log 400 - \log 16 \)
   d. \( \log \left( \frac{1}{400} \right) + 2 \log 2 \)

6. **Mathematical Reflection** Explain why \( \log (-100) \) is not defined.
Exponential Functions and Common Logarithms

WHETHER OR NOT

1. Tell whether or not each table contains data that can be modeled by an exponential function. Provide an equation to show the relationship between $x$ and $y$ for the sets of data that are exponential.

   a. 
   \[
   \begin{array}{c|c|c|c|c}
   x & 0 & 1 & 2 & 3 \\
   \hline
   y & 3 & 6 & 12 & 24 \\
   \end{array}
   \]

   b. 
   \[
   \begin{array}{c|c|c|c|c}
   x & 0 & 1 & 2 & 3 \\
   \hline
   y & 2 & 4 & 6 & 8 \\
   \end{array}
   \]

   c. 
   \[
   \begin{array}{c|c|c|c|c}
   x & 0 & 1 & 2 & 3 \\
   \hline
   y & 108 & 36 & 12 & 4 \\
   \end{array}
   \]

2. Tell whether or not each function is increasing. State increasing or decreasing, and give the $y$-intercept.

   a. $y = 4 \left( \frac{2}{3} \right)^x$  
   b. $y = -3(4)^x$

3. Let $g(x) = 2(4)^{x+3} - 5$.

   a. Describe the function as a transformation of $y = 4^x$.

   b. Graph the function on a separate sheet of grid paper, using your knowledge of transformations.

   c. What is the horizontal asymptote of the graph of $g$?

4. Evaluate each expression without using a calculator.

   a. $\log 1000$  
   b. $\log 1$  
   c. $\log \frac{1}{10,000}$

   d. $10^{\log 8}$  
   e. $\log 2 + \log 50$  
   f. $\log 4 - \log 400$

   g. $3 \log 5 + \log 8$  
   h. $\log 10^{-3}$
### Exponential Functions and Common Logarithms

**WHETHER OR NOT**

<table>
<thead>
<tr>
<th>Math Knowledge #2, 3c, 4</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student:</td>
<td>The student:</td>
<td></td>
<td>The student:</td>
</tr>
<tr>
<td>- Correctly identifies the increasing or decreasing function(s) and gives the correct y-intercept for the function(s). (2)</td>
<td>- Correctly identifies the increasing or decreasing function(s) or gives the correct y-intercept for the functions.</td>
<td>- Neither identifies the increasing or decreasing function(s) nor gives the correct y-intercept for the functions.</td>
<td></td>
</tr>
<tr>
<td>- Gives the correct horizontal asymptote of the graph of $g$. (3c)</td>
<td>- Evaluates at least five expressions correctly.</td>
<td>- Gives the incorrect horizontal asymptote of the graph of $g$.</td>
<td></td>
</tr>
<tr>
<td>- Evaluates all eight expressions correctly. (4)</td>
<td></td>
<td>- Evaluates at least two expressions correctly.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem Solving #1a, b, c</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student tells which table contains data that can be modeled by an exponential function. (1a, b, c)</td>
<td>The student identifies only two tables correctly.</td>
<td>The student identifies only one table correctly.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Representations #1a, b, c; 3b</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student:</td>
<td>The student:</td>
<td></td>
<td>The student:</td>
</tr>
<tr>
<td>- Writes correct exponential function(s) for the data given. (1a, b, c)</td>
<td>- Identifies some, but not all three tables correctly.</td>
<td>- Identifies none of the tables correctly.</td>
<td></td>
</tr>
<tr>
<td>- Graphs $g(x)$ correctly. (3b)</td>
<td></td>
<td>- Graphs $g(x)$ incorrectly.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Communication #3a</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student gives a complete, accurate description of $g(x)$ as a transformation of $y = 4^x$. (3a)</td>
<td>The student gives an incomplete description of $g(x)$ as a transformation of $y = 4^x$; the explanation contains no mathematical errors.</td>
<td>The student gives a description that contains mathematical errors.</td>
<td></td>
</tr>
</tbody>
</table>
In the last unit, you studied inverses of linear functions. Recall that two functions \(f\) and \(g\) are inverses of each other if and only if \(f(g(x)) = x\) for all \(x\) in the domain of \(g\), and \(g(f(x)) = x\) for all \(x\) in the domain of \(f\).

1. Find the inverse \(g(x)\) of the function \(f(x) = 2x + 1\). Show your work.

2. Use the definition of inverse functions to prove that \(f(x) = 2x + 1\) and the \(g(x)\) function you found in Item 1 are inverse functions.

3. Graph \(f(x) = 2x + 1\) and its inverse on the grid below. What is the line of symmetry between the graphs?

In Activity 2.4, you investigated exponential functions with a base of 10 and their inverse functions, the common logarithmic functions. Recall in the Richter scale situation that \(G(x) = 10^x\), where \(x\) is the magnitude of an earthquake. The inverse function is \(M(x) = \log x\), where \(x\) is the ground motion compared to a standard earthquake.
4. A part of each of the graphs of \( y = G(x) \) and \( y = M(x) \) is shown below. What is the line of symmetry between the graphs? How does that line compare with the line of symmetry in Item 3?

![Graphs of G(x) and M(x)]

Logarithms with bases other than 10 have the same properties as common logarithms.

The logarithm of \( y \) with base \( b \), where \( y > 0, b > 0, b \neq 1 \), is defined as:

\[
\log_b y = x \text{ if and only if } y = b^x
\]

The exponential function \( y = b^x \) and the logarithmic function \( y = \log_b x \), where \( b > 0 \) and \( b \neq 1 \), are inverse functions.

5. Let \( g(x) = f^{-1}(x) \), the inverse of function \( f \). Write the rule for \( g \) for each function \( f \) given below.

a. \( f(x) = 5^x \)  
b. \( f(x) = \log_b x \)
6. Use the functions from Item 5. Complete the expression for each composition.

   a. \( f(x) = 5^x \)
      
      \[ f(g(x)) = \quad = x \]
      
      \[ g(f(x)) = \quad = x \]

   b. \( f(x) = \log_4 x \)
      
      \[ f(g(x)) = \quad = x \]
      
      \[ g(f(x)) = \quad = x \]

7. Use what you learned in Item 6 to complete these inverse properties for logarithms. Assume \( b > 0 \) and \( b \neq 1 \).

   a. \( b^{\log_b x} = \quad \)
   
   b. \( \log_b b^x = \quad \)

8. Simplify each expression.

   a. \( 6^{\log_6 x} \)
   
   b. \( \log_3 3^x \)

   c. \( 8^{\log_8 x} \)
   
   d. \( \log 10^x \)

When expressing statements between exponential and logarithmic form, it is helpful to remember that a logarithm is an exponent. The exponential statement \( 2^3 = 8 \) is equivalent to the logarithmic statement \( \log_2 8 = 3 \). Notice that the logarithmic expression is equal to 3, which is the exponent in the exponential expression.

9. Express each exponential statement as a logarithmic statement.

   a. \( 3^4 = 81 \)
   
   b. \( 6^{-2} = \frac{1}{36} \)
10. Express each logarithmic statement as an exponential statement.
   a. $\log_{4} 16 = 2$
   b. $\log_{3} 125 = 3$

11. Evaluate each expression without using a calculator.
   a. $\log_{2} 32$
   b. $\log_{4} \left( \frac{1}{64} \right)$
   c. $\log_{3} 27$
   d. $\log_{12} 1$

The Product, Quotient, and Power Properties for common logarithms also extend to bases other than base 10.

12. Use the given property to condense each expression to a single logarithm. Then evaluate each logarithm in the equation to see that both sides of the equation are equal.
   a. Product Property: $\log_{2} 4 + \log_{2} 8 = \ldots$
      
   b. Quotient Property: $\log_{3} 27 - \log_{3} 3 = \ldots$
   c. Power Property: $2 \log_{5} 25 = \ldots$

13. Expand each expression. Assume all variables are positive.
   a. $\log_{3} \frac{x}{y^3}$
   b. $\log_{4} x^2 y$
14. Assume that \( x \) is any real number and decide whether the statement is
always true, sometimes true, or never true. If the statement is
sometimes true, give the conditions for which it is true.

a. \( \log 7 - \log 5 = \frac{\log 7}{\log 5} \)

b. \( \log_5 5^x = x \)

c. \( 2^{\log_2 x^2} = x^2 \)

d. \( \log_4 3 + \log_4 5 - \log_4 x = \log_4 15 \)

Sometimes it is useful to change the base of a logarithmic expression. For
example, changing the base makes it easier to work with logarithms on a
calculator.

15. Use the common logarithm function on a calculator to find the
numerical value of each expression. Write the value in the first
column of the table. Then write the numerical value using logarithms
in base 2 in the second column.

<table>
<thead>
<tr>
<th>Numerical Value</th>
<th>( \log_2 a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\log 2}{\log 2} )</td>
<td>1</td>
</tr>
<tr>
<td>( \frac{\log 4}{\log 2} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{\log 8}{\log 2} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{\log 16}{\log 2} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{\log N}{\log 2} )</td>
<td></td>
</tr>
</tbody>
</table>

16. The patterns observed in the table in Item 15 illustrate the Change of
Base Formula. Make a conjecture about the Change of Base Property
of logarithms.

\( \log_b x = \) ___________
17. Consider the expression $\log_2 12$.

a. The value of $\log_2 12$ lies between which two integers?

b. Write an equivalent expression for $\log_2 12$, using the Change of Base Formula.

c. Use a calculator to find the value of $\log_2 12$, correct to three decimal places. Compare the value to your answer to Part (a).

18. Examine the function $f(x) = 2^x$ and its inverse $g(x) = \log_2 x$.

a. Complete the table of data for $f(x) = 2^x$. Then use that data to complete a table of values for $g(x) = \log_2 x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = 2^x$</th>
<th>$x$</th>
<th>$g(x) = \log_2 x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>−1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Graph both $f(x) = 2^x$ and $g(x) = \log_2 x$ on the grid in the My Notes space.

c. What is the line of symmetry between the graphs of $f(x) = 2^x$ and $g(x) = \log_2 x$?

d. State the domain and range of each function.

<table>
<thead>
<tr>
<th></th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = 2^x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g(x) = \log_2 x$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Inverse Functions

Undoing It All

**SUGGESTED LEARNING STRATEGIES:** Close Reading, Activating Prior Knowledge, Create Representations, Quickwrite

18. (continued)

   e. Write the equation of any asymptotes of each function.

   \[ f(x) = 2^x \quad \text{and} \quad g(x) = \log_2 x \]

Transformations of the graph of the function \( f(x) = \log_b x \) can be used to graph functions of the form \( g(x) = a \log_b (x - c) + d \), where \( b > 0, b \neq 1 \). You can draw a quick sketch of each parent graph, \( f(x) = \log_b x \), by plotting the points \( \left( \frac{1}{b}, -1 \right), (1, 0) \) and \( (b, 1) \).

19. Sketch the parent graph \( f(x) = \log_b x \) on the axes below. Then for each transformation of \( f \), provide a verbal description and sketch the graph including asymptotes.

   a. \( g(x) = 3 \log_2 x \)

   b. \( h(x) = 3 \log_2 (x + 4) \)

   c. \( j(x) = 3 \log_2 (x + 4) - 2 \)

**Math Tip**

Recall that a graph of the exponential function \( f(x) = b^x \) can be drawn by plotting the points \( \left( -1, \frac{1}{b} \right), (0, 1) \) and \( (1, b) \). Switching the \( x \) and \( y \)-coordinates of these points gives you three points on the graph of the inverse of \( f(x) = b^x \), which is \( f(x) = \log_b x \).
20. Explain how the function \( j(x) = 3 \log_2(x + 4) - 2 \) can be entered on a graphing calculator. Then graph the function on a calculator and compare the graph to your answer in Item 19c.

21. Explain how the parameters \( a, c, \) and \( d \) transform the parent graph \( f(x) = \log_b x \) to produce a graph of the function \( g(x) = a \log_b(x - c) + d \).
Wesley is researching college costs. He is considering two schools: a four-year private college where tuition and fees for the current year cost about $24,000, and a four-year public university where tuition and fees for the current year cost about $10,000. Wesley learned that the last decade, tuition and fees have increased an average of 5.6% per year in four-year private colleges and an average of 7.1% per year in four-year public colleges.

To answer Items 1–4, assume that tuition and fees continue to increase at the same average rate per year as in the last decade.

1. Complete the table of values to show the estimated tuition for the next four years.

<table>
<thead>
<tr>
<th>Years From Present</th>
<th>Private College Tuition And Fees</th>
<th>Public College Tuition And Fees</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$24,000</td>
<td>$10,000</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Write two functions to model the data in the table above. Let \( R(t) \) represent the private tuition and fees and \( U(t) \) represent the public tuition and fees, where \( t \) is the number of years from the present.

3. Wesley plans to be a senior in college six years from now. Use the models above to find the estimated tuition and fees at both the private and public colleges for his senior year in college.
### Activity 2.6 (continued)

**College Costs**

**SUGGESTED LEARNING STRATEGIES:** Create Representations, Vocabulary Organizer, Note Taking, Group Presentation

4. Write an equation that can be solved to predict the number of years that it will take for the public college tuition and fees to reach the current private tuition and fees of $24,000. Find the solution using both the graphing and table features of a calculator.

Solving a problem like the one in Item 4 involves solving an exponential equation. An exponential equation is an equation in which the variable is in the exponent. Sometimes you can solve an exponential equation by writing both sides of the equation in terms of the same base. Then use the fact that when the bases are the same, the exponents must be equal:

\[ b^m = b^n \text{ if and only if } m = n \]

**EXAMPLE 1**

Solve \( 6 \cdot 4^x = 96 \).

\[
\begin{align*}
6 \cdot 4^x &= 96 \\
\text{Step 1} & \quad 4^x = 16 \\
\text{Step 2} & \quad 4^x = 4^2 \\
\text{Step 3} & \quad x = 2 \\
\end{align*}
\]

**EXAMPLE 2**

Solve \( 5^{4x} = 125^{x-1} \).

\[
\begin{align*}
5^{4x} &= 125^{x-1} \\
\text{Step 1} & \quad (5^3)^x = (5^3)^{x-1} \\
\text{Step 2} & \quad 5^{4x} = 5^{3x-3} \\
\text{Step 3} & \quad 4x = 3x - 3 \\
\text{Step 4} & \quad x = -3 \\
\end{align*}
\]

**TRY THESE A**

Solve for \( x \). Write your answers in the My Notes space. Show your work.

\[
\begin{align*}
a. \quad 3^x - 1 &= 80 \\
b. \quad 2^x &= \frac{1}{32} \\
c. \quad 6^{3x-4} &= 36^{x+1}
\end{align*}
\]
For many exponential equations, it is not possible to rewrite the equation in terms of the same base. In this case, use the concept of inverses to solve the equation symbolically.

**EXAMPLE 3**

Estimate the solution of $3^x = 32$. Then solve to three decimal places.

Estimate that $x$ is between 3 and 4, because $3^3 = 27$ and $3^4 = 81$.

- **Step 1** $\log_3 3^x = \log_3 32$  
  *Take the log base 3 of both sides.*
- **Step 2** $x = \log_3 32$  
  *Use the inverse property to simplify the left side.*
- **Step 3** $x = \frac{\log 32}{\log 3}$  
  *Use the Change of Base Formula.*
- **Step 4** $x \approx 3.155$  
  *Use a calculator to simplify.*

**TRY THESE B**

Estimate each solution. Then solve to three decimal places. Write your answers in the My Notes space. Show your work.

- **a.** $6^x = 12$
- **b.** $5^x = 610$
- **c.** $4^x = 0.28$

**EXAMPLE 4**

Find the solution of $4^{x-2} = 35.6$ to three decimal places.

- **Step 1** $\log_4 4^{x-2} = \log_4 35.6$  
  *Take the log base 4 of both sides.*
- **Step 2** $x - 2 = \log_4 35.6$  
  *Use the inverse property to simplify the left side.*
- **Step 3** $x = \frac{\log 35.6}{\log 4} + 2$  
  *Use the Change of Base Formula.*
- **Step 4** $x \approx 4.577$  
  *Use a calculator to simplify.*

**TRY THESE C**

Find each solution to three decimal places. Write your answers in the My Notes space. Show your work.

- **a.** $12^{x+3} = 240$
- **b.** $4.2^{x+4} + 0.8 = 5.7$
5. Show how to solve the equation you wrote in Item 4, using the inverse property.

Wesley’s grandfather gave him a birthday gift of $3000 to use for college. Wesley plans to deposit the money in a savings account. He can use the formula below to find the amount of money in his savings account after a given period of time.

**Compounded Interest Formula**

\[ A = P \left( 1 + \frac{r}{n} \right)^{nt} \]

- \( A \) = amount in account
- \( P \) = principal invested
- \( r \) = annual interest rate
- \( n \) = number of times per year that interest is compounded
- \( t \) = number of years

6. If Wesley deposits the gift from his grandfather into an account that pays 4% annual interest compounded quarterly, how much money will Wesley have in the account after 3 years?
ACTIVITY 2.6 continued

College Costs

SUGGESTED LEARNING STRATEGIES: Create Representations, Vocabulary Organizer, Note Taking

7. How long would it take an investment of $5000 to earn $1000 interest if it is invested in a savings account that pays 3.75% annual interest compounded monthly?

Equations that involve logarithms of variable expressions are called logarithmic equations. You can solve some logarithmic equations symbolically by using the concept of functions and their inverses. Since the domain of logarithmic functions is restricted to the positive real numbers, it is necessary to check for extraneous solutions when solving logarithmic equations.

EXAMPLE 5
Solve log₄ (3x - 1) = 2.

\[
\log_4 (3x - 1) = 2
\]

Step 1 \[4^{\log_4 (3x - 1)} = 4^2\] Write in exponential form using 4 as the base.

Step 2 \[3x - 1 = 16\] Use the inverse property to simplify the left side.

Step 3 \[x = \frac{17}{3}\] Solve for x.

Check: \[\log_4 (3 \cdot \frac{17}{3} - 1) = \log_4 16 = 2\]

To solve other logarithmic equations use the fact that when the bases are the same and \(b, m, n > 0, b \neq 1\), the logarithmic values must be equal:

\[\log_b m = \log_b n \text{ if and only if } m = n\]
EXAMPLE 6
Solve \( \log_3(2x - 3) = \log_3(x + 4) \).

\( \log_3(2x - 3) = \log_3(x + 4) \)

**Step 1** \( 2x - 3 = x + 4 \) \( \text{If } \log_b m = \log_b n, \text{ then } m = n. \)

**Step 2** \( x = 7 \) \( \text{Solve for } x. \)

Check:

\( \log_3(2 \cdot 7 - 3) \triangleq \log_3(7 + 4) \)

\( \log_3 11 = \log_3 11 \)

TRY THESE D
Solve for \( x \). Check for extraneous solutions. Write your answers in the My Notes space. Show your work.

a. \( \log_6 (3x + 4) = 1 \)

b. \( \log_5 (7x - 2) = \log_5 (3x + 6) \)

Sometimes it is necessary to use properties of logarithms to simplify one side of a logarithmic equation before solving the equation.

EXAMPLE 7
Solve \( \log_2 x + \log_2 (x + 2) = 3 \).

\( \log_2 x + \log_2 (x + 2) = 3 \)

**Step 1** \( \log_2 [x(x + 2)] = 3 \) \( \text{Product Property of Logarithms} \)

**Step 2** \( 2^{\log_2 [x(x + 2)]} = 2^3 \) \( \text{Write in exponential form using } 2 \text{ as the base.} \)

**Step 3** \( x(x + 2) = 8 \) \( \text{Use the inverse property to simplify.} \)

**Step 4** \( x^2 + 2x - 8 = 0 \) \( \text{Write as a quadratic equation.} \)

**Step 5** \( (x + 4)(x - 2) = 0 \) \( \text{Solve the quadratic equation.} \)

**Step 6** \( x = -4 \text{ or } x = 2 \) \( \text{Check for extraneous solutions.} \)

Check:

\( \log_2 (-4) + \log (-4 + 2) \triangleq 3 \)

\( \log_2 (-4) + \log (-2) \triangleq 3 \)

\( \log_2 2 + \log (2 + 2) \triangleq 3 \)

\( \log_2 2 + \log 4 \triangleq 3 \)

\( \log_2 8 \triangleq 3 \)

\( \frac{3}{3} = 3 \)

Because \( \log_2 (-4) \) and \( \log (-2) \) are not defined, \( -4 \) is not a solution of the original equation; thus it is extraneous.

The solution is \( x = 2. \)
EXAMPLE 8

Solve \( \log_2 (x^2 + 2x) = 3 \).

\[ \log_2 (x^2 + 2x) = 3 \]  

**Step 1**  
\[ 2^{\log_2 (x^2 + 2x)} = 2^3 \]  
Write in exponential form using 2 as the base.

**Step 2**  
\[ x^2 + 2x = 8 \]  
Use the inverse property to simplify.

**Step 3**  
\[ x^2 + 2x - 8 = 0 \]  
Write as a quadratic equation.

**Step 4**  
\[ (x + 4)(x - 2) = 0 \]  
Solve the quadratic equation.

**Step 5**  
\[ x = -4 \text{ or } x = 2 \]  
Check for extraneous solutions.

Check:  
\[ \log_2 [(-4)^2 + 2(-4)] \stackrel{?}{=} 3 \]  
\[ \log_2 [2^2 + 2(2)] \stackrel{?}{=} 3 \]  
\[ \log_2 8 \stackrel{?}{=} 3 \]  
\[ \log_2 8 \stackrel{?}{=} 3 \]  
\[ 3 = 3 \]  
\[ 3 = 3 \]

The solutions are \( x = -4 \) or \( x = 2 \).

TRY THESE E

Solve for \( x \). Check for extraneous solutions. Write your answers in the My Notes space. Show your work.

a. \( \log_8 4 + \log_8 (x + 1) = 2 \)  
b. \( \log_4 x = 2 - \log_4 (x + 6) \)

c. \( \log (x + 1) - \log x = 1 \)

Some problems can be solved using exponential or logarithmic inequalities. Write an inequality, and then use a calculator to solve the inequality graphically.

8. Scientists have found a relationship between atmospheric pressure and altitudes up to 50 miles above sea level that can be modeled by.  
\[ f(x) = 14.7(0.5)^{\frac{x}{3}} \]  
The value \( f(x) \) is the atmospheric pressure in lb/in.\(^2\) and \( x \) is altitude in miles above sea level. Find the altitude above sea level when the atmospheric pressure is less than 5 lb/in.\(^2\).
9. The relationship between the price of an item and the quantity supplied is often analyzed. Suppose that the relationship between \( f(x) \), the number of digital cameras supplied, and the price \( x \) per camera in dollars is modeled by the function \( f(x) = -400 + 180 \cdot \log x \). Find the range in the price predicted by the model if there are between 20 and 30 cameras supplied.

**CHECK YOUR UNDERSTANDING**

Write your answers on notebook paper. Show your work.

1. Solve for \( x \) by writing both sides of the equation in terms of the same base.
   a. \( 2^{10x} = 32 \)
   b. \( 4^x - 5 = 11 \)
   c. \( 2^{4x-2} = 4^{x+2} \)
   d. \( 8^x = \frac{1}{64} \)

2. Solve for \( x \), to three decimal places.
   a. \( 8^x = 100 \)
   b. \( 3^{x-4} = 85 \)
   c. \( 2^{3x-2} + 7 = 25 \)
   d. \( 2 \cdot 4^{3x} - 3 = 27 \)

3. A deposit of $4000 is made into a savings account that pay 2.48% annual interest compounded quarterly.
   a. How much money will be in the account after 3 years?
   b. How long will it take for the account to earn $500 interest?

4. Solve for \( x \). Check for extraneous solutions.
   a. \( \log_3 (3x + 4) = 2 \)
   b. \( \log_3 (4x + 1) = 4 \)
   c. \( \log_{12} (4x - 2) = \log_{12} (x + 10) \)
   d. \( \log_2 3 + \log_2 (x - 4) = 4 \)
   e. \( \log_3 (x + 4) - \log_3 (x - 4) = 4 \)
   f. \( \log_4 (x + 6) - \log_4 x = 2 \)

5. Use a graphing calculator to solve each inequality.
   a. \( 16.4(0.87)^{x-1.5} \geq 10 \)
   b. \( 30 < 25 \log (3.5x - 4) + 12.6 < 50 \)

6. **MATHEMATICAL REFLECTION** How is it possible to have more than one solution to a simplified logarithmic equation, only one of which is valid?
Exponential and Logarithmic Equations

EVALUATING YOUR INTEREST

1. Evaluate each expression without using a calculator.
   a. \(25^{\log_5 x}\)  
   b. \(\log_3 3^x\)  
   c. \(\log_5 27\)
   d. \(\log_8 1\)  
   e. \(\log_2 40 - \log_2 5\)  
   f. \(\frac{\log 25}{\log 5}\)

2. Solve each equation symbolically. Give approximate answers correct to three decimal places. Check the solutions. Write your answers on notebook paper. Show your work.
   a. \(4^{2x-1} = 64\)  
   b. \(5^x = 38\)
   c. \(3^{x+2} = 98.7\)  
   d. \(2^{3x-4} + 7.5 = 23.6\)
   e. \(\log_3 (2x + 1) = 4\)  
   f. \(\log_8 (3x - 2) = \log_8 (x + 1)\)
   g. \(\log_2 (3x - 2) + \log_2 8 = 5\)  
   h. \(\log_6 (x - 5) + \log_6 x = 2\)

3. Let \(f(x) = \log_2 (x - 1) + 3\).
   a. Use a separate sheet of grid paper to sketch a parent graph and a series of transformations that result in the graph of \(f\).
   b. Give the equation of the vertical asymptote of graph of \(f\).

4. How long would it take an investment of $6500 to earn $1200 interest if it is invested in a savings account that pays 4% annual interest compounded quarterly? Show the solution both graphically on grid paper and symbolically below.
## Exponential and Logarithmic Equations

### EVALUATING YOUR INTEREST

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<thead>
<tr>
<th>Math Knowledge #1a-f, 2a-h</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
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<td>The student:</td>
<td>The student:</td>
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</tr>
<tr>
<td>• Evaluates all six expressions correctly, using the appropriate properties of logarithms when necessary. (1a-f)</td>
<td>• Evaluates at least four expressions correctly.</td>
<td>• Evaluates at least two expressions correctly.</td>
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<tr>
<td>• Solves all eight equations correctly to three decimal places. (2a-h)</td>
<td>• Solves between at least five equations correctly. OR • Solves all eight correctly, but does not round them correctly.</td>
<td>• Solves at least two equations correctly.</td>
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<tr>
<td>• Identifies the correct parent graph. (3a)</td>
<td>• Does not identify the correct graph.</td>
<td></td>
</tr>
<tr>
<td>• Gives the correct number of years. (4)</td>
<td>• Gives the incorrect number of years.</td>
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<td>The student:</td>
<td></td>
</tr>
<tr>
<td>• Sketches a correct parent graph and correct graphs of all of the transformations. (3a)</td>
<td>• Sketches a correct parent graph and correct graphs for some of the transformations.</td>
<td>• Sketches a correct parent graph; graphs for the transformations are either incorrect or missing.</td>
<td></td>
</tr>
<tr>
<td>• Writes the correct equation of the vertical asymptote of the graph of ( f ). (3b)</td>
<td>• Writes the correct equation of the vertical asymptote of an incorrect graph of ( f ).</td>
<td>• Writes an incorrect equation of the vertical asymptote of the graph drawn.</td>
<td></td>
</tr>
<tr>
<td>• Gives a symbolic solution with no mathematical errors and draws a correct graph of the solution. (4)</td>
<td>• Gives either a correct symbolic solution or a correct graph of the solution.</td>
<td>• Gives neither a correct symbolic solution nor a correct graph of the solution.</td>
<td></td>
</tr>
</tbody>
</table>
ACTIVITY 2.1

1. Determine whether or not each sequence is arithmetic. If the sequence is arithmetic, state the common difference.
   a. 4, 12, 20, 28, …
   b. 5, 10, 20, 40, …
   c. 4, 0, −4, −8
   d. 6, 7, 8, 9, …

2. Find the indicated term of each arithmetic sequence.
   a. \(a_1 = -2, \ d = 4; \ a_{12}\)
   b. 15, 19, 23, 27, …; \(a_{10}\)
   c. 46, 40, 34, 28, …; \(a_{20}\)

3. Find the indicated partial sum of each arithmetic series.
   a. \(a_1 = -3, \ d = 4; \ S_{10}\)
   b. 26 + 24 + 22 + 20 + …; \(S_{12}\)
   c. \[\sum_{n=1}^{20} (2n + 1)\]

4. A radio station offers a $100 prize on the first day of a contest. Each day that the prize money is not awarded, $50 is added to the prize amount. If a contestant wins on the 17th day of the contest, how much money will be awarded?

ACTIVITY 2.2

5. Write arithmetic, geometric, or neither for each sequence. If arithmetic, state the common difference. If geometric, state the common ratio.
   a. 4, 12, 36, 108, 324, …
   b. 1, 2, 6, 24, 120, …
   c. 4, 9, 14, 19, 24, …
   d. 35, −30, 25, −20, 15, …

6. Find the indicated term of each geometric series.
   a. \(a_1 = 1, \ r = -3; \ a_{10}\)
   b. \(a_1 = 3072, \ r = \frac{1}{4}; \ a_8\)

7. Find the indicated partial sum of each geometric series.
   a. 5, 2, \(\frac{4}{3}, \frac{8}{9}, \ldots; \ S_{\infty}\)
   b. \[\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, \ldots; \ S_{15}\]

8. In case of a school closing due to inclement weather, the high school staff has a calling system to make certain that everyone is notified. In the first round of phone calls, the principal calls three staff members. In the second round of calls, each of those three staff members calls three more staff members. The process continues until all of the staff is notified.
   a. Write a rule that shows how many staff members are called during the \(n\)th round of calls.
   b. Find the number of staff members called during the 4th round of calls.
   c. If all of the staff has been notified after the 4th round of calls, how many people are on staff at the high school, including the principal?

9. Find the infinite sum if it exists. If it does not exist, tell why.
   a. \(24 + 12 + 6 + 3 + \ldots\)
   b. \[\frac{1}{12} + \frac{1}{6} + \frac{1}{3} + \frac{2}{3} + \ldots\]
   c. \(1296 - 216 + 36 - 6 + \ldots\)

ACTIVITY 2.3

10. Decide whether each table of data can be modeled by a linear function, an exponential function, or neither. If the data can be modeled by a linear or exponential function, give an equation for the function.
   a. 
   
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<tbody>
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<td>1</td>
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<td>27</td>
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   b. 
   
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   c. 
   
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<td>12</td>
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   d. 
   
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<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>36</td>
<td>18</td>
<td>9</td>
<td>4.5</td>
<td>2.25</td>
</tr>
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</table>
11. The World Factbook produced by the Central Intelligence Agency estimates the July 2008 United States population as 303,824,640. The following rates are also reported as estimates for 2008.

**Birth rate:** 14.18 births/1000 population  
**Death rate:** 8.27 deaths/1000 population  
**Net migration rate:** 2.92 migrant(s)/1000 population

a. Write a percent for each rate listed above.

b. Combine the percents from part (a) to find the overall growth rate for the United States.

c. The exponential growth factor for a population is equal to the growth rate plus 100%. What is the exponential growth rate for the United States?

d. Write a function to express the United States population as a function of years since 2008.

e. Use the function from part d to predict the United States population in the year 2050.

12. Describe how each function results from transforming a parent graph of the form \( f(x) = b^x \). Then sketch the parent graph and the given function on the same axes. State the domain and range of each function and give the equations of any asymptotes.

a. \( g(x) = 4^{-x^2} - 3 \)

b. \( g(x) = \frac{1}{2}(2)^{-x^4} + 1 \)

c. \( g(x) = -2\left(\frac{1}{3}\right)^{x^4} \)

**ACTIVITY 2.4**

13. What function has a graph that is symmetric to the graph of \( y = \log x \) about the line \( y = x \)?

14. Write an exponential statement for each logarithmic statement below.

a. \( \log 10,000 = 4 \)

b. \( \log \frac{1}{1,000,000,000} = -9 \)

c. \( \log a = 6 \)

**ACTIVITY 2.5**

15. Write a logarithmic statement for each exponential statement below.

\( a. \ 10^{-2} = \frac{1}{100} \quad b. \ 10^1 = 10 \quad c. \ 10^4 = n \)

16. Evaluate without using a calculator.

\( a. \ \log 10^5 \quad b. \ 10^{\log 7} \quad c. \ \log 100 \quad d. \ \log \frac{1}{100,000} \)

17. Complete each statement to illustrate a property for logarithms.

\( a. \ \text{Product Property} \quad \log uv = ? \quad b. \ \text{Quotient Property} \quad \log \frac{u}{v} = ? \quad c. \ \text{Power Property} \quad \log u^r = ? \)

18. Condense each expression to a single logarithm. Then evaluate without using a calculator.

\( a. \ \log 500 + \log 2 \quad b. \ 2 \log 3 + \log \frac{1}{9} \quad c. \ \log 80 - 3 \log 2 \)

19. Expand each expression.

\( a. \ \log xy^2 \quad b. \ \log \frac{xy}{2} \quad c. \ \log a^4b^2 \)

20. Give the inverse for each function.

\( a. \ f(x) = \left(\frac{1}{3}\right)^x \quad b. \ f(x) = \log_{20} x \)

21. Simplify each expression.

\( a. \ 12^\log_{12} x \quad b. \ \log_7 7^x \)

22. Write a logarithmic statement for each of the following exponential statements.

\( a. \ 2^3 = 144 \quad b. \ 2^{-3} = \frac{1}{8} \)

23. Write an exponential statement for each of the following logarithmic statements.

\( a. \ \log_3 27 = -3 \quad b. \ \log_6 216 = 3 \)
24. Evaluate each expression without using a calculator.
   a. \( \log_2 64 \)  
   b. \( \log_4 256 \)  
   c. \( \log_{13} 169 \)  
   d. \( \log_4 \left( \frac{1}{16} \right) \)

25. Use logarithm properties to expand each expression. Assume all variables are positive.
   a. \( \log_2 x^2 y^5 \)  
   b. \( \log_4 \left( \frac{x^8}{5} \right) \)

26. Use logarithm properties to condense each expression to a single logarithm. Assume all variables are positive.
   a. \( \log a - 4 \log b \)  
   b. \( \frac{\log(x)}{\log(6)} \)

27. Change each logarithmic expression to a logarithmic expression in base 10. Then use a calculator to evaluate the expression to three decimal places.
   a. \( \log_7 130 \)  
   b. \( \log_5 5 \)

28. Graph each function, using a parent graph and the appropriate transformations. Describe the transformations.
   a. \( f(x) = \frac{1}{2} \log_4 (x) \)  
   b. \( f(x) = \log_4 (x + 3) - 4 \)

ACTIVITY 2.6

29. Solve for \( x \) by writing both sides of the equation in terms of the same base.
   a. \( 8 \cdot 3^x = 216 \)  
   b. \( 5^x = \frac{1}{625} \)  
   c. \( 7^{2x} = 343^{x-4} \)  
   d. \( 4^x + 8 = 72 \)

30. Solve for \( x \) to three decimal places.
   a. \( 7^x = 300 \)  
   b. \( 5^{x-4} = 135 \)  
   c. \( 3^{2x+1} - 5 = 80 \)  
   d. \( 3 \cdot 6^{x-0.01} = 0.38 \)

31. A deposit of $1000 is made into a savings account that pay 4% annual interest compounded monthly.
   a. How much money will be in the account after 6 years?
   b. How long will it take for the $1000 to double?

32. Solve for \( x \). Check for extraneous solutions.
   a. \( \log_5 (5x - 2) = 3 \)  
   b. \( \log_4 (2x - 3) = 2 \)  
   c. \( \log_7 (5x + 3) = \log_7 (3x + 11) \)  
   d. \( \log_4 4 + \log_8 (x + 2) = 1 \)  
   e. \( \log_5 (x + 8) = 2 - \log_5 (x) \)  
   f. \( \log_3 (x + 6) - \log_2 (x) = 3 \)

33. Use a graphing calculator to solve each inequality.
   a. \( 2000 < 1500(1.04)^{12x} < 3000 \)  
   b. \( 4.5 \log(2x) + 8.4 \geq 9.2 \)
An important aspect of growing as a learner is to take the time to reflect on your learning. It is important to think about where you started, what you have accomplished, what helped you learn, and how you will apply your new knowledge in the future. Use notebook paper to record your thinking on the following topics and to identify evidence of your learning.

**Essential Questions**

1. Review the mathematical concepts and your work in this unit before you write thoughtful responses to the questions below. Support your responses with specific examples from concepts and activities in the unit.
   - How are functions that grow at a constant rate distinguished from those that do not grow at a constant rate?
   - How are logarithmic and exponential equations used to model real-world problems?

**Academic Vocabulary**

2. Look at the following academic vocabulary words:
   - exponential function
   - increasing/decreasing
   - sequence
   - extraneous solution
   - logarithm
   - series
   Choose three words and explain your understanding of each word and why each is important in your study of math.

**Self-Evaluation**

3. Look through the activities and Embedded Assessments in this unit. Use a table similar to the one below to list three major concepts in this unit and to rate your understanding of each.

<table>
<thead>
<tr>
<th>Unit Concepts</th>
<th>Is Your Understanding Strong (S) or Weak (W)?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concept 1</td>
<td></td>
</tr>
<tr>
<td>Concept 2</td>
<td></td>
</tr>
<tr>
<td>Concept 3</td>
<td></td>
</tr>
</tbody>
</table>

a. What will you do to address each weakness?
b. What strategies or class activities were particularly helpful in learning the concepts you identified as strengths? Give examples to explain.

4. How do the concepts you learned in this unit relate to other math concepts and to the use of mathematics in the real world?
1. Which expression is equivalent to $\log 20 + \log 2$?
   A. $\log \frac{20}{2}$
   B. $\log 22$
   C. $\log 40$
   D. $\log 80$

2. Which term in the arithmetic sequence 7, 12, 17, 22, … has a value of 132?

3. A new car costs $33,000. The value of the car depreciates (decreases) by 18% each year. What is the value (to the nearest dollar) of the car after 3 years?
4. Given the function \( f(x) = 2^x + 1 \):

   **Part A:** Give the domain, range, \( y \)-intercept, and any asymptotes for \( f(x) \).

   **Answer and Explain**

   

   

   **Part B:** Draw a sketch of the graph of the function on the grid below. Describe the behavior of the function as \( x \) approaches \( \infty \) and as \( x \) approaches \(-\infty \).

   **Answer and Explain**

   

   

   

   

   ![Graph Grid]